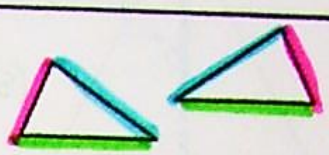
TOPIC 4: THREE METHODS OF PROOF (SSS, SAS, ASA)

There are 5 ways to prove two triangles congruent.

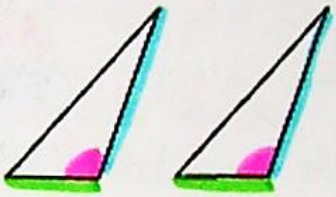
Below you will learn the first 3 methods.

1. **Side-Side-Side:** If the 3 sides of one triangle are congruent to the 3 sides of another triangle, then the triangles are congruent.



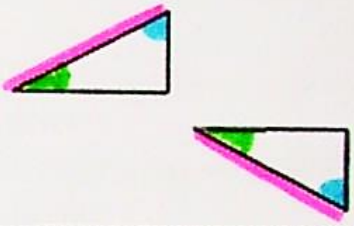
Abbreviation: SSS

2. **Side-Angle-Side:** If 2 sides and the included angle of a triangle are congruent to 2 sides and the included angle of another triangle, then the triangles are congruent.



Abbreviation: SAS

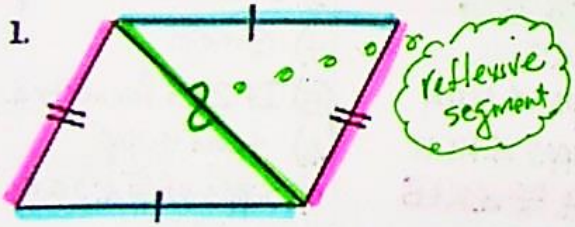
3. **Angle-Side-Angle:** If 2 angles and the included side of a triangle are congruent to 2 angles and the included side of another triangle, then the triangles are congruent.



Abbreviation: ASA

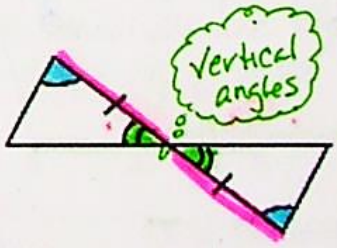
Determine if each pair of triangles can be proved congruent. If they can be, state which method can be used. If they cannot be, write "*Cannot be proved*".

1.



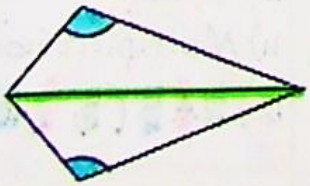
Yes! by SSS

2.



Yes! by ASA

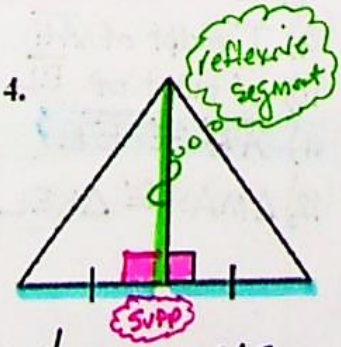
3.



Cannot be proved

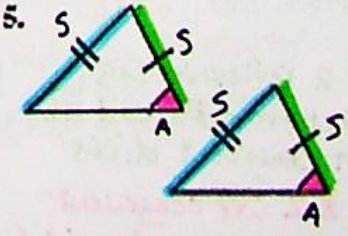
SA[?]

4.



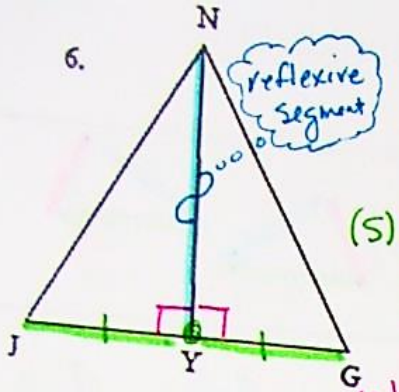
Yes! by SAS

5.



NO! Angle not included, so cannot be proved

6.

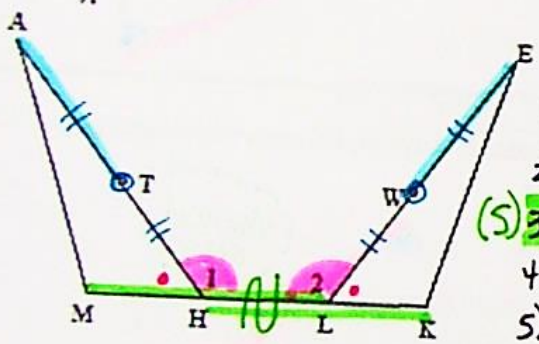


Given: Y midpt $\overline{JG}$
$\overline{NY} \perp \overline{JG}$

Prove: $\Delta NYJ \cong \Delta NYG$

Statements	Reasons
1) Y mdpt $\overline{JG}$	1) Given
(S) 2) $\overline{JY} \cong \overline{YG}$	2) A mdpt ÷ seg into 2 $\cong$ segs
3) $\overline{NY} \perp \overline{JG}$	3) Given
4) $\angle NYJ$ is a Rt $\angle$ $\angle NYG$ is a Rt $\angle$	4) $\perp$ segs form Rt $\angle$'s
(A) 5) $\angle NYJ \cong \angle NYG$	5) All Rt $\angle$'s are $\cong$
(S) 6) $\overline{NY} \cong \overline{NY}$	6) Reflexive Property
7) $\Delta NYJ \cong \Delta NYG$	7) SAS (2, 5, 6)

7.



Given: $\overline{ML} \cong \overline{HK}$
$\angle 1 \cong \angle 2$
○ T midpt $\overline{AH}$
○ W midpt $\overline{EL}$
$\overline{AT} \cong \overline{EW}$

Prove: $\Delta MAH \cong \Delta KEL$

Statements	Reasons
1) $\overline{ML} \cong \overline{HK}$	1) Given
2) $\overline{HL} \cong \overline{HL}$	2) Reflexive Prop.
(S) 3) $\overline{MH} \cong \overline{LK}$	3) Subtraction Prop.
4) $\angle 1 \cong \angle 2$	4) Given
5) $\angle 1$ supps $\angle MHA$	5) If 2 $\angle$'s form str $\angle$,
6) $\angle 2$ supps $\angle KLE$	6) Hon supp
(A) 7) $\angle MHA \cong \angle KLE$	7) supps of $\cong \angle$'s are $\cong$
8) $\overline{AT} \cong \overline{EW}$	8) Given
9) T mdpt of $\overline{AH}$	9) Given
10) W mdpt of $\overline{EL}$	10) Given
(S) 11) $\overline{AH} \cong \overline{EL}$	11) Multiplication Prop.
12) $\Delta MAH \cong \Delta KEL$	12) SAS (3, 7, 11)

- $\overline{HL}$ is a reflexive seg, subtract from $\overline{ML}$ and $\overline{HK}$ to get congruent diffs
- $\angle 1$ & $\angle 2$ are congruent & they supp an interior $\angle$ in each Δ.
- Since T and W are mdpts and $\overline{AT} \cong \overline{EW}$, we can double (multiply) to get longer $\cong$ segments

Name: _____

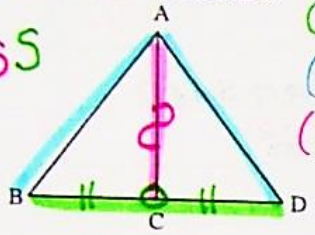
Geometry Worksheet: 3.2 3 Ways to Prove Two Triangle Congruent Date: _____

At this point, to prove two triangles are congruent, you will need 3 congruencies:

SSS or SAS or ASA

1. Given: C mdpt. $\overline{BD}$
   $\overline{AB} \cong \overline{AD}$

   Prove: $\Delta ABC \cong \Delta ADC$

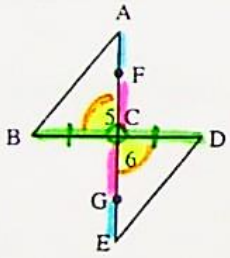


SSS

Statements	Reasons
1) C mdpt of $\overline{BD}$	1) Given
(S) 2) $\overline{BC} \cong \overline{CD}$	2) A mdpt ÷ seg into 2 $\cong$ segs
(S) 3) $\overline{AB} \cong \overline{AD}$	3) Given
(S) 4) $\overline{AC} \cong \overline{AC}$	4) Reflexive Property
5) $\Delta ABC \cong \Delta ADC$	5) SSS (2, 3, 4)

2. Given: C mdpt $\overline{BD}$
   $\overline{AF} \cong \overline{GE}$
   $\overline{FC} \cong \overline{CG}$

   Prove: $\Delta ACB \cong \Delta ECD$

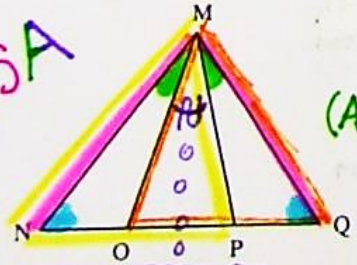


SAS

Statements	Reasons
1) C mdpt of $\overline{BD}$	1) Given
(S) 2) $\overline{BC} \cong \overline{CD}$	2) A mdpt ÷ seg into 2 $\cong$ segs
3) $\overline{AF} \cong \overline{GE}$	3) Given
4) $\overline{FC} \cong \overline{CG}$	4) Given
(S) 5) $\overline{AC} \cong \overline{CE}$	5) Addition Property
(A) 6) $\angle 5 \cong \angle 6$	6) Vertical $\angle$'s are $\cong$
7) $\Delta ACB \cong \Delta ECD$	7) SAS (2, 6, 5)

3. Given: $\angle N \cong \angle Q$
   $\overline{MN} \cong \overline{MQ}$
   $\angle NMO \cong \angle QMP$

   Prove: $\Delta NMP \cong \Delta QMO$



ASA

reflexive angle $\angle OMP$

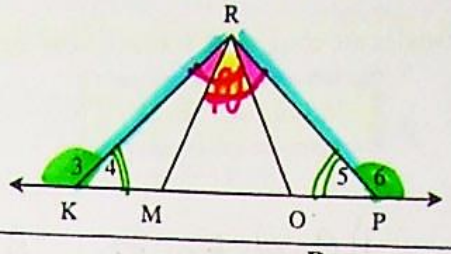
Statements	Reasons
(A) 1) $\angle N \cong \angle Q$	1) Given
(S) 2) $\overline{MN} \cong \overline{MQ}$	2) Given
3) $\angle NMO \cong \angle QMP$	3) Given
4) $\angle OPM \cong \angle OPM$	4) Reflexive Property
(A) 5) $\angle NMP \cong \angle QMO$	5) Addition Property
6) $\Delta NMP \cong \Delta QMO$	6) ASA (1, 2, 5)

Geometry WS#2 Date:

3.2 Proving Triangles Congruent

Given: $\angle 3 \cong \angle 6$
$\overline{KR} \cong \overline{PR}$
$\angle KRO \cong \angle PRM$

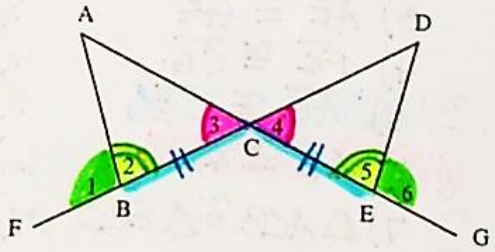
Prove: $\triangle KRM \cong \triangle PRO$



Statements	Reasons
1. $\angle 3 \cong \angle 6$	1. Given
2. $\angle 3$ supps $\angle 4$, $\angle 6$ supps $\angle 5$	2. If 2 $\angle$'s form a str. $\angle$, then supp
(A) 3. $\angle 4 \cong \angle 5$	3. Supps of $\cong$ $\angle$'s are $\cong$
(S) 4. $\overline{KR} \cong \overline{PR}$	4. Given
5. $\angle KRO \cong \angle PRM$	5. Given
6. $\angle MRO \cong \angle MRO$	6. Reflexive Property
(A) 7. $\triangle KRM \cong \triangle PRO$	7. Subtraction Property
8. $\triangle KRM \cong \triangle PRO$	8. ASA (3, 4, 7)

2. Given: $\angle 1 \cong \angle 6$
$\overline{BC} \cong \overline{EC}$

Prove: $\triangle ABC \cong \triangle DEC$

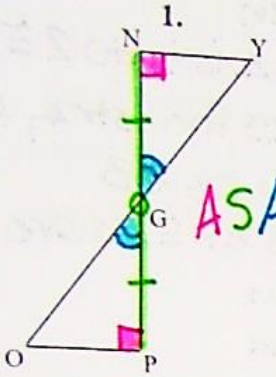


Statements	Reasons
1. $\angle 1 \cong \angle 6$	1. Given
2. $\angle 1$ supps $\angle 2$	2. If 2 $\angle$'s form str $\angle$, then supp
3. $\angle 6$ supps $\angle 5$	3. Same as #2
(A) 4. $\angle 2 \cong \angle 5$	4. Supps of $\cong$ $\angle$'s are $\cong$
(S) 5. $\overline{BC} \cong \overline{EC}$	5. Given
6. $\angle 3$ & $\angle 4$ vert. $\angle$s	6. Assumed from diagram
(A) 7. $\angle 3 \cong \angle 4$	7. Vertical $\angle$'s are $\cong$
8. $\triangle ABC \cong \triangle DEC$	8. ASA (4, 5, 7)

Geometry: 3.2 Proof Practice!

Name _____

1.



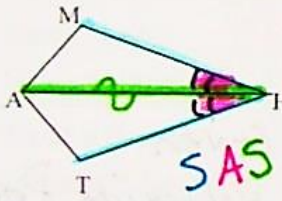
ASA

Given: G midpt. $\overline{NP}$; $\overline{NY} \perp \overline{PN}$; $\overline{OP} \perp \overline{PN}$

Prove: $\Delta NYG \cong \Delta POG$

Statements	Reasons
1) G mdpt of NP	1) Given
(S) 2) $\overline{NG} \cong \overline{GP}$	2) A mdpt ÷ seg into 2 ≅ segs
3) $\overline{PN} \perp \overline{NY}$ & $\overline{OP}$	3) Given
4) $\angle N$ & $\angle P$ are Rt $\angle$'s	4) ⊥ segs form Rt $\angle$'s
(A) 5) $\angle N \cong \angle P$	5) All right $\angle$'s are ≅
6) $\angle NGY$ & $\angle PGO$ are vertical $\angle$'s	6) Assumed from diagram
(A) 7) $\angle NGY \cong \angle PGO$	7) Vertical $\angle$s are ≅
8) $\Delta NYG \cong \Delta POG$	8) ASA (5, 2, 7)

2.



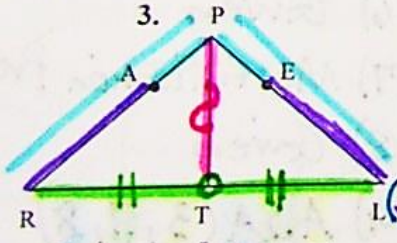
SAS

Given: $\overrightarrow{HA}$ bis. $\angle MHT$; $\overline{MH} \cong \overline{TH}$

Prove: $\Delta HAM \cong \Delta HAT$

Statements	Reasons
1) $\overrightarrow{HA}$ bis $\angle MHT$	1) Given
(A) 2) $\angle MHA \cong \angle THA$	2) A bis $\angle$ is ÷ into 2 ≅ $\angle$'s
(S) 3) $\overline{MH} \cong \overline{TH}$	3) Given
(S) 4) $\overline{AH} \cong \overline{AH}$	4) Reflexive Property
5) $\Delta HAM \cong \Delta HAT$	5) SAS (3, 2, 4)

3.



SSS

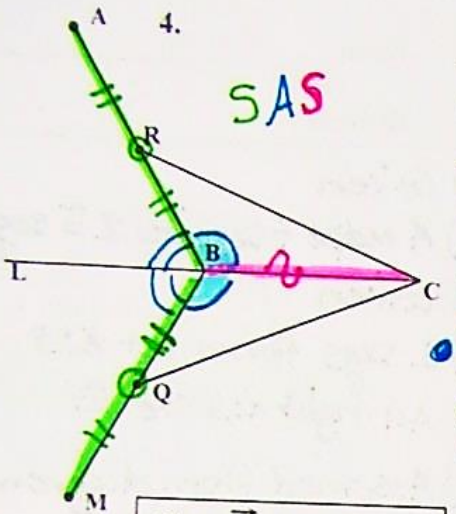
Given: $\overline{PA} \cong \overline{PE}$; $\overline{AR} \cong \overline{EL}$; T midpt. $\overline{RL}$

Prove: $\Delta PRT \cong \Delta PLT$

Statements	Reasons
1) $\overline{PA} \cong \overline{PE}$	1) Given
2) $\overline{AR} \cong \overline{EL}$	2) Given
(S) 3) $\overline{PR} \cong \overline{PL}$	3) Addition Prop.
4) T mdpt of $\overline{RL}$	4) Given
(S) 5) $\overline{RT} \cong \overline{TL}$	5) A mdpt ÷ seg into 2 ≅ segs
(S) 6) $\overline{PT} \cong \overline{PT}$	6) Reflexive Property
7) $\Delta PRT \cong \Delta PLT$	7) SSS (3, 5, 6)

4.

SAS



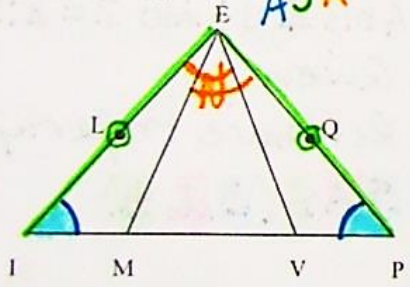
Given: $\overrightarrow{BL}$ bis. $\angle RBQ$
R midpt. $\overline{AB}$
Q midpt. $\overline{MB}$
$\overline{AB} \cong \overline{MB}$

Prove: $\Delta RBC \cong \Delta QBC$

Statements	Reasons
1) $\overrightarrow{BL}$ bis $\angle RBQ$	1) Given
2) $\angle RBL \cong \angle QBL$	2) A bis $\angle$ is $\div$ into $2 \cong \angle$'s
3) $\angle RBL$ supps $\angle RBC$	3) If 2 $\angle$'s form str $\angle$, then s
4) $\angle QBL$ supps $\angle QBC$	4) Same as #3
5) $\angle RBC \cong \angle QBC$ (A)	5) supps of $\cong \angle$s are $\cong$
6) R mdpt of $\overline{AB}$	6) Given
7) Q mdpt of $\overline{MB}$	7) Given
8) $\overline{AB} \cong \overline{MB}$	8) Given
9) $\overline{RB} \cong \overline{QB}$ (S)	9) Division Property
10) $\overline{BC} \cong \overline{BC}$ (S)	10) Reflexive Property
11) $\Delta RBC \cong \Delta QBC$	11) SAS (9, 5, 10)

5.

ASA



Given: $\angle IEV \cong \angle PEM$
L midpt. $\overline{EI}$
Q midpt. $\overline{EP}$
$\overline{EL} \cong \overline{PQ}$
$\angle I \cong \angle P$

Prove: $\Delta EIM \cong \Delta EPV$

Statements	Reasons
1) $\angle IEV \cong \angle PEM$	1) Given
2) $\angle MEV \cong \angle MEV$	2) Reflexive Property
3) $\angle IEM \cong \angle PEV$ (A)	3) Subtraction Prop.
4) L mdpt $\overline{EI}$	4) Given
5) Q mdpt $\overline{EP}$	5) Given
6) $\overline{EL} \cong \overline{PQ}$	6) Given
7) $\overline{EI} \cong \overline{EP}$ (S)	7) Multiplication Prop.
8) $\angle I \cong \angle P$ (A)	8) Given
9) $\Delta EIM \cong \Delta EPV$	9) ASA (3, 7, 8)